C. Ancient Philosophy; 3rd Student Essay

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Analyze in detail any one of zeno's paradoxes.

Let us for a moment imagine we had the ability to travel back in time. Also let us not bother how this strange state of affairs came about or its possibility, but let us just suppose we can. It's around 5th c. B.C.E. The place is Athens and we are at the festival of the Great Panathenaea. The main event of the day is the race between Achilles, legendary Greek hero and the tortoise. The tortoise had challenged Achilles and our good friend Zeno the Eleatic had been kind enough to arrange the race. The terms of the race agreed to by both participants were thus; the tortoise would be given a head start of some distance and that Achilles should not be considered the winner unless he could overtake the tortoise. Achilles being the son of the sea-goddess Thetis thought this terms very reasonable, and at the sound of Zeno's gong the two racers were off. Let's suppose the starting distance between Achilles and the tortoise is 50 meters. When the super-fast feet of Achilles arrived at the point from which the tortoise started, of course the tortoise was no longer there. Let's suppose the tortoise has moved a distance of about 5 meters. Achilles swiftly covers these 10 meters but the tortoise has moved again. Time has elapsed; looking at both runners it seems like Achilles is ahead. But the fact is that since both runners were moving, whenever Achilles reaches somewhere the tortoise had been he still has farther to go. Of course this distance would keep reducing ad infinitum.

To understand this better let us imagine two points A and B. Achilles is at A and the tortoise is at B. Before Achilles can cover the entire distance he must go halfway. But before he can go halfway he must cover a quarter of the distance. But before that, one eighth, such that we can conceive of ever smaller distances till infinity. So before Achilles can even start from his original position he would have to exhaust an infinite series. But the problem is an infinite series is inexhaustible. For instance it is impossible to count from one to infinity for it is an infinite series. In essence if space (this includes the space between Achilles and the tortoise) and time were infinitely divisible then motion would be impossible.

But is space infinitely divisible? Are the objects of our experience infinitely divisible? Bringing it closer home am I infinitely divisible such that I could be composed of infinitely many smaller parts? A good distinction to make at this point that would enable us to have a deeper insight into this matter is the one raised by Aristotle. He made the distinction between potential and actual infinity. An actual infinity is something which is complete and definite and consists of infinitely many elements; no new members can be added to it. While potential infinity is something that is never complete: more and more elements can always be added, but never infinitely many. If space and the objects of our experience are infinitely divisible it would mean the existence of an actual infinite. In the same vein, Achilles cannot overtake the tortoise because the infinite series he has to transverse is an actual one and not a potential one.

But does an actual infinite exist? In modern mathematics the use of actually infinite sets is commonplace. For example the set of the natural numbers (0,1,2,3,4,…..) has an actually infinite number of members in it. Although we may be able to mentally conceive of an actual infinite as it is used in mathematics, this does not show that such mathematical entities really exist or that an actually infinite number of things can really exist. For if an actually infinite number of things did exist it would lead to absurdities. For instance imagine a blackberry factory with an actual infinite number of black and white phones. If someone broke in and took all the white phones (just imagine the possibility of stealing an infinite number of phones, when will the thief be done?) the factory will still have the same (infinite) number of phones left. But this flies straight in the face of our intuitions for we know that part of something must be less than the whole, so if all the white phones go missing how we can still have the same number of phones. Based on these considerations the existence of an actual infinite seems impossible.

But is there a way the real world could have been that it is possible for there to be an actual infinite? Or put differently is there some possible world in which there exists an actual infinite? Logical possibility is a prerequisite for something to exist in a possible world, and logical possibility is simply that it does not entail a logical contradiction. For example one plus one equals four is a logically impossible state of affairs because it entails a logical contradiction. Hence there is no possible world in which it can obtain. Is this the case with an actual infinite? While there might be some possible world in which there is an actual infinite the fact remains that such a world would consist of a conundrum of contradictory situations and absurdities.

So what? replies our good friend Zeno who helped to arrange the race. That is the whole point of the race, to show us that Achilles cannot overtake the tortoise. Yes, motion is also impossible along with so many other absurdities primarily because we have put too much stock in how things appear to us rather than how they really are which can only be gotten by the dictates of reason and not the senses. What is clear is that the problem of infinites goes beyond the issue of there being a plurality of things or just a single entity. Maybe the issue is not there being one or many things but the nature of things. For if things were basically immaterial in their essences but appear to be material the problem of infinites would not come up. For something that is immaterial is not composed of parts, so how do you divide something that is not composed of parts? Well, it didn't take an infinitely long time to tell this story or did it?